



## KNOW YOUR STUFF

### Understanding Odds in Lottery Games

Washington's Lottery players enjoy six different games which depend on the luck of the draw to determine the winning numbers. Players select a set of numbers, or let the computer select for them, and Washington's Lottery randomly draws the winning numbers. If the player's numbers match the numbers drawn, the player wins!

For a simple tabulation of the odds of winning, see the prize tables for each game on the Lottery Web site. To learn how these odds are calculated, keep reading. This is a quick explanation of how the probability of winning games of this kind can be **understood**. It is meant to be intuitive and fun since it is based on actual Lottery games. For a more formal explanation, see a math textbook that has a section on probability, or search online sources using the search words "permutation," "combinatorial," or related terms.

### The Games

| [Mega Millions](#) | [Lotto](#) | [Hit 5](#) | [Match 4](#) | [The Daily Game](#) | [Daily Keno](#) |

Each of the six Lottery draw games offers different odds and different ways to play and win, but for calculating odds, there are two distinct game types.

For five of the six Lottery games, it's the numbers that matter, and the order they are drawn is irrelevant. But for one game, The Daily Game, both the numbers and the particular order they are picked, matters. A winning ticket must match the numbers drawn, **in the order they are drawn**.

#### Common Terms

**Draw** – The random selection of a number by the Lottery

**Pick** – The selection of a number by a player

**The field** – The range of possible numbers which can be drawn

**Independent event** – Two events are independent if the fact that one occurs does not change the likelihood of the other occurring



## When Order Matters – The Daily Game

In The Daily Game (known as Pick 3 in some other states) the field is zero through nine (10 items). There are three draws, and each time the starting field is zero through nine (think of the drawn number being “replaced” before the next draw). These events are independent. In other words, the odds are the same for each draw. With 10 different numbers, the probability of drawing any one number is 1 in 10 (1/10, or 0.1). The second and third draws are independent of the first and have the same probability, 1/10.

The probability of multiple independent events happening in combination is the product of their individual probabilities. The player has a 1/10 chance of matching on each of the three draws, so the odds of matching all three is the product of the probability for each draw.

So a player’s odds of winning the jackpot in The Daily Game is  $1/10 * 1/10 * 1/10$  or **1 in 1,000**.

Once a player picks a number it is a fact, not a possibility: its probability is 1. The likelihood that the Lottery will pick the same number is 1 in 1000.

Additionally, in The Daily Game it is possible to bet on only part of the draw: “front pair,” “back pair,” and so on. The odds are calculated in exactly the same way as above, but players have a better chance to win (a lower prize).

Players who bet on the front pair win if their numbers match the first two numbers drawn. Calculating the probability is the same as above, but only the first two draws matter. This means that probability for matching the first two numbers is  $1/10 * 1/10$  or 1/100. The odds of winning a front pair bet is 1 in 100.

## When Order Does Not Matter

For games where the order of the numbers drawn doesn’t matter, calculating the odds becomes a bit more complicated. Match 4, Hit 5, Lotto, Mega Millions, and Daily Keno are all games where the order does not matter.

### Match 4

For this example, we’ll look at Match 4. In this game, the field for the first draw is one through 24. But unlike The Daily Game, in Match 4 the starting field of numbers changes at each draw because the previously-drawn numbers are not replaced. Once a number is drawn, it can’t be drawn a second time.

This means that the first draw has 24 possible outcomes (one through 24); and this leaves 23 remaining outcomes for the second draw, 22 remaining outcomes for the third draw and 21 outcomes for the fourth and final draw.

Combining independent events, the probability of drawing any particular set of four numbers in order is:  $1/24 * 1/23 * 1/22 * 1/21 = 1/255,024$ .



In Match 4, we have said that order does not matter. Thus among the quarter-million different possible “picks”, there are many equivalent rearrangements of the same four numbers.

If you were to rearrange four numbers, the first number of the arrangement can be any of the four numbers. The second number can be any number except the one that was already chosen. The third number can be any number except the two already chosen and the last is the remaining number.

This means there are  $(4 \times 3 \times 2 \times 1 =)$  24 ways of arranging four numbers.

Consequently, there are  $(255,024 / 24 =)$  10,626 possible four number combinations in Match 4. Each one has equal probability of being selected. The player picks one, and the Lottery may match it, with probability **1 in 10,626**.

#### **Lower Prize When Order Does Not Matter**

The jackpot isn't the only prize players can win. Many games define lower prizes when a player's pick matches part of the Lottery's draw. For instance, in Match 4, there is a prize for matching three out of four numbers.

Let's consider this as “missing” one of the Lottery-drawn numbers. How many ways are there to “miss”? First of all, there are four numbers that can be “missed”. How many ways are there to pick a “wrong” number? Recall that we already picked four from 24, so there are 20 left. There are 20 ways to be wrong on each of four choices, for a total of 80 ways to miss one number. Consequently, a player is 80 times as likely to miss just one number as to match all four. Stated another way, the likelihood of matching three out of four is 1 in  $(10,626 / 80 =)$  132.825.

Further, in Match 4, there is a prize for matching only two of four numbers. We can think of this as “missing two numbers”. There are six number pairs that can be “missed” (six ways to take four things two at a time). There are 20 “wrong” numbers. There are 20 ways to pick the first “wrong” number, and 19 ways to pick the second “wrong” number: this gives  $(20 \times 19 =)$  380 ways to pick two wrong numbers. In Match 4, we do not care about the order of occurrences, and clearly there are two equivalent ways of getting each pair of wrong numbers, so this leaves  $(380/2 =)$  190 different combinations of two wrong numbers. Each of these 190 “wrong” combinations can substitute for any of the six combinations of two “right” numbers. Consequently, there are  $(190 \times 6 =)$  1140 ways of missing two numbers. Missing exactly two numbers is thus 1140 times as likely as matching all four numbers. Stated another way, the likelihood of matching two out of four numbers is 1 in  $(10,626 / 1140 =)$  9.32.

#### **Overall Odds**

The likelihood of any win in a game is described by “overall odds.” “Overall odds” are the sum of the probabilities of all the independent winning events. In Match 4, we have described three ways to win. We express the probability of each way as a decimal (one divided by the odds) in order to add them up:



Match	Odds - 1 in	Probability
4 of 4	10,626	.000094109
3 of 4	132.82	.007529
2 of 4	9.32	.107284
Any win		.114907

In other words, plays in Match 4 win about 11 percent of the time. Usually, we convert the decimal back to “odds” form when talking about the game:

Match 4 overall odds are  $(1 / 0.114907 =) 1$  in 8.703.

#### **Other Games Where Order Does Not Matter**

Hit 5, Lotto, Mega Millions, and Daily Keno are all games in which the order in which numbers are picked does not matter. The odds of their various outcomes can be understood in the same way as described here. However, because these games use bigger fields and have a number of different prize tiers, most people use a strictly formal mathematical approach to calculating their odds. The formal approach simply expresses the reasoning described here in mathematical formulas. The formal approach does not introduce any new ideas beyond those discussed above.

#### **Congratulations!**

If you understand what you have read on these pages, you understand far more about Lottery games than most people! Using the techniques explained above, you can calculate the odds of any of Washington’s Lottery games.